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Volkov-Akulov theory and D-branes[1]

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ABSTRACT

The action of supersymmetric Born-Infeld theory (D-9-brane in a Lorentz covariant static gauge) has a geometric form of the Volkov-Akulov-type. The first non-linearly realized supersymmetry can be made manifest, the second world-volume supersymmetry is not manifest. We also study the analogous 2 supersymmetries of the quadratic action of the covariantly quantized D-0-brane. We show that the Hamiltonian and the BRST operator are build from these two supersymmetry generators.

This article is dedicated to the memory of D. V. Volkov whose insights into the nature of supersymmetry and geometry proved to be enlightening for few generations of high-energy physicists. His ideas inspired the most active developments in theoretical physics over the last quarter of the century.

Extended objects with global supersymmetry have local κ-symmetry. This symmetry is difficult to quantize in Lorentz covariant gauges keeping finite number of fields in the theory. A revival of interest to κ-symmetric objects is due to the recent discovery of D-p-branes [1], κ-symmetric non-linear effective actions and/or equations of motion for D-branes [2, 3, 4, 5, 6, 7] and the M-5-brane

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action [8, 9, 10], complementing the κ-symmetric superstring and M-2-brane [11, 12]. A nice review on worldvolume actions in a doubly supersymmetric geometric approach initiated by D. V. Volkov is presented in these Proceedings [13].

The new issues in quantization of D-branes have been analyzed recently in [3, 7, 14].

The κ-symmetric D-brane action in the flat background geometry[3] consists of the Born-Infeld-Nambu-Goto term S_1 and Wess-Zumino term S_2:

$$S_{\rm DBI} + S_{\rm WZ} = T\left(-\int d^{p+1}\sigma\,\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu})} + \int \Omega_{p+1}\right) \ . \tag{1}$$

Here T is the tension of the D-brane, $G_{\mu\nu}$ is the manifestly supersymmetric induced world-volume metric

$$G_{\mu\nu} = \eta_{mn}\Pi^m_\mu \Pi^n_\nu \ , \qquad \Pi^m_\mu = \partial_\mu X^m - \bar\theta\Gamma^m\partial_\mu\theta \ , \tag{2}$$

and $\mathcal{F}_{\mu\nu}$ is a manifestly supersymmetric Born-Infeld field strength (for p even) [4]

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - b_{\mu\nu} = \Big[\partial_\mu A_\nu - \bar\theta\Gamma_{11}\Gamma_m\partial_\mu\theta\Big(\partial_\nu X^m - \frac{1}{2}\bar\theta\Gamma^m\partial_\nu\theta\Big)\Big] - (\mu\leftrightarrow\nu) \ . \tag{3}$$

When p is odd, Γ_{11} is replaced by $\tau_3\otimes I$. The action has global supersymmetry

$$\delta_\epsilon\theta = \epsilon, \qquad \delta_\epsilon X^m = \bar\epsilon\Gamma^m\theta \ . \tag{4}$$

and local κ-supersymmetry:

$$\delta X^m = \bar\theta\Gamma^m\delta\theta = -\delta\bar\theta\Gamma^m\theta, \qquad \delta\bar\theta = \bar\kappa(1+\Gamma), \tag{5}$$

and

$$\Gamma = e^{\frac{a}{2}}\Gamma'_{(0)}e^{-\frac{a}{2}} \ , \qquad a = \begin{cases} +\frac{1}{2}Y_{jk}\gamma^{jk}\Gamma_{11} & \text{IIA} \ , \\ -\frac{1}{2}Y_{jk}\gamma^{jk}\sigma_3\otimes 1 & \text{IIB} \ . \end{cases} \tag{6}$$

Here $\Gamma'_{(0)}$ is the product structure, independent on the BI field, $(\Gamma'_{(0)})^2 = 1, tr\,\Gamma'_{(0)} = 0)$. All dependence on BI field $\mathcal{F} =$ "tan"Y is in the exponent [7]. The matrix Γ_{11} in IIA and $\sigma_3\otimes\ 1$ in IIB theory anticommute with $\Gamma'_{(0)}$ and with Γ. Therefore in the basis where Γ_{11} and $\sigma_3\otimes\ 1$ are diagonal, $\Gamma'_{(0)}$ and Γ are off-diagonal. We introduce a 16×16-dimensional matrix $\hat\gamma$ which does not depend on BI field.

$$\Gamma'_{(0)} = \begin{pmatrix} 0 & \hat\gamma \\ \hat\gamma^{-1} & 0 \end{pmatrix} \ , \qquad \Gamma = \begin{pmatrix} 0 & \hat\gamma e^{\hat a} \\ (\hat\gamma e^{\hat a})^{-1} & 0 \end{pmatrix} \ , \tag{7}$$

where

$$\hat a = \begin{cases} +\frac{1}{2}Y_{jk}\gamma^{jk} & \text{IIA} \ , \\ -\frac{1}{2}Y_{jk}\gamma^{jk} & \text{IIB} \ . \end{cases} \tag{8}$$

[3]We use notation of [3].

[4]We define spinors for even p as $\theta = \theta_1 + \theta_2$ where $\theta_1 \equiv \frac{1}{2}(1+\Gamma_{11})\theta$ and $\theta_2 \equiv \frac{1}{2}(1-\Gamma_{11})\theta$.

The fact that Γ is off-diagonal and that the matrix $\gamma e^{\hat{a}}$ is invertible is quite important and the significance of this for covariant quantization of D-branes was already discussed in [3, 7, 14]. In particular this allows us to consider only irreducible κ-symmetry transformations by imposing a Lorentz covariant condition on $\bar{\kappa}$ of the form

$$\bar{\kappa}_1 = 0 \qquad \bar{\kappa}_2 \neq 0 \qquad \text{IIA} \tag{9}$$

$$\bar{\kappa}_2 = 0 \qquad \bar{\kappa}_1 \neq 0 \qquad \text{IIB} \ . \tag{10}$$

In this way we have an irreducible 16-dimensional κ-symmetry since the matrix $\hat{\gamma} e^{\hat{a}}$ is invertible, acting as

$$\delta\bar{\theta}_1 = \bar{\kappa}_2 \hat{\gamma} e^{\hat{a}} \qquad \delta\bar{\theta}_2 = \bar{\kappa}_2 \qquad \delta X^m = -\bar{\kappa}_2 \Gamma^m \theta_2 \qquad \text{IIA} \tag{11}$$

$$\delta\bar{\theta}_1 = \bar{\kappa}_1 \qquad \delta\bar{\theta}_2 = \bar{\kappa}_1 (\hat{\gamma} e^{\hat{a}})^{-1} \qquad \delta X^m = -\bar{\kappa}_1 \Gamma^m \theta_1 \qquad \text{IIB} \ . \tag{12}$$

Recently a covariant gauge fixing κ-symmetry of D-branes has been discovered [3]. The fermionic gauge is of the form

$$\theta_2 = 0 \qquad \text{IIA} \qquad \theta_1 \equiv \lambda \tag{13}$$

$$\theta_1 = 0 \qquad \text{IIB} \qquad \theta_2 \equiv \lambda \ . \tag{14}$$

Note that our choice of irreducible κ-symmetry (which is not unique) was made here with the purpose to explicitly eliminate θ_2 (θ_1) in IIA (IIB) case using $\delta\bar{\theta}_2 = \bar{\kappa}_2$ ($\delta\bar{\theta}_1 = \bar{\kappa}_1$). The gauge-fixed action has one particularly useful property: the Wess-Zumino term vanishes in this gauge [3]. We are left with the reparametrization invariant action:

$$S_{\kappa-\text{fixed}} = -\int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} \ , \tag{15}$$

$$G_{\mu\nu} = \eta_{mn} \Pi^m_\mu \Pi^n_\nu \ , \qquad \Pi^m_\mu = \partial_\mu X^m - \bar{\lambda} \Gamma^m \partial_\mu \lambda \ , \tag{16}$$

$$\mathcal{F}_{\mu\nu} = [\partial_\mu A_\nu - \bar{\lambda} \Gamma_m \partial_\mu \lambda \left(\partial_\nu X^m - \frac{1}{2} \bar{\lambda} \Gamma^m \partial_\nu \lambda \right)] - (\mu \leftrightarrow \nu) \ . \tag{17}$$

This action (18) has a local reparametrization symmetry and a 32-component global supersymmetry. The form of the action is such that it can be brought to the form close to the one discovered by Volkov-Akulov [15]. Consider for example a D-9-brane in a static gauge [3] $X^\mu = \sigma^\mu$:

$$S_{g.f.} = -\int d^{10}\sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} \ , \tag{18}$$

where

$$G_{\mu\nu} = e_\mu{}^m e_\nu{}^n \eta^{mn} \tag{19}$$

and

$$e_\mu{}^m = \delta_\mu{}^m - \bar{\lambda} \Gamma^m \partial_\mu \lambda \ . \tag{20}$$

If we introduce the 1-forms depending on fermion fields $\lambda(\sigma)$

$$e^m = d\sigma^\mu e_\mu{}^m[\lambda(\sigma)] = d\sigma^m + \bar\lambda\Gamma^m d\lambda \tag{21}$$

we can rewrite the supersymmetric Born-Infeld 9-brane action as

$$S = \int e^{m_0}\wedge e^{m_1}\wedge\cdots\wedge e^{m_9}\sqrt{-\det(\eta_{mn}+\mathcal{F}_{mn})} \tag{22}$$

where $\mathcal{F}_{mn} = e^\mu{}_m e^\nu{}_n \mathcal{F}_{\mu\nu}$. In absence of the two-form $\mathcal{F}_{mn}$ the supersymmetric Born-Infeld action is reduced to geometric action of Volkov-Akulov [15], generalized to d=10:

$$S = \int e^{m_0}\wedge e^{m_1}\wedge\cdots\wedge e^{m_9} = \int d^{10}\sigma \, \det e[\lambda(\sigma)] \tag{23}$$

This action depends only on fermions $\lambda(\sigma)$ and has a non-linearly realized supersymmetry manifest, since the 1-forms e^m are supersymmetric. The second supersymmetry of the supersymmetric Born-Infeld 9-brane action (22) is not manifest. It explicit form can be obtained from the preservation of the gauge-fixing condition for the kappa-symmetry.

The gauge-fixed actions of extended supersymmetric objects in static gauge are known to lead to complicated non-linear actions. For example, the action of d=2 massive superparticle [16]

$$S = -M\int dt\left(\left[-(\dot X^m - \bar\theta\Gamma^m\dot\theta)(\dot X^n - \bar\theta\Gamma^n\dot\theta)\eta_{mn}\right]^{1/2} - 1 + \theta\Gamma^3\dot\theta\right) \qquad m=0,1. \tag{24}$$

quantized in the gauge $\Gamma^3\theta = \theta$ for κ-symmetry and the static gauge for reparametrization symmetry, $X^0 = t$, gives the kink effective action [16]

$$S_{g.f} = -M\int dt\left(\left[1-\dot\phi^2\right]^{1/2} - 1 + \frac{i}{4M}\rho\dot\rho\right) \ . \tag{25}$$

Here the bosonic field is the remaining coordinate of the d=2 superparticle, $\phi = X^1$. The Hamiltonian associated with this action is also non-linear:

$$H = (p^2+M^2)^{1/2} - M \ , \tag{26}$$

p is the momentum canonical conjugate to X^1.

Here we will perform a covariant quantization of the D-0-brane which is a d=10 generalization of the d=2 massive superparticle [16]. Instead of the static gauge, which belongs to a class of canonical gauges with the non-propagating ghosts, we will use a covariant gauge for reparametrization symmetry. Consider the κ-symmetric action of a D-0-brane [2, 3, 4, 5, 6]. D-0-brane action does not have Born-Infeld field since there is no place for an antisymmetric tensor of rank 2 in one-dimensional theory. The D-p-brane action for $p=0$ case reduces to

$$S = -T\left(\int d\tau\sqrt{-G_{\tau\tau}} + \int \bar\theta\Gamma^{11}\dot\theta\right) \ . \tag{27}$$

This action can be derived from the action of the massless 11-dimensional superparticle [4].

$$S = \int d\tau \sqrt{-g_{\tau\tau}} g^{\tau\tau} \left(\dot{X}^{\hat{m}} - \bar{\theta}\Gamma^{\hat{m}}\dot{\theta} \right)^2 , \qquad \hat{m} = 0, 1, \cdots, 8, 9, 10. \tag{28}$$

We may solve equation of motion for $X^{\hat{10}}$ as $\mathbf{P}_{\hat{10}} = Z$, where Z is a constant, and use $\Gamma^{11} = \Gamma^{\hat{10}}$. From this one can deduce a first order action

$$S = \int d\tau \left(\mathbf{P}_m(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta}) + \frac{1}{2}V(\mathbf{P}^2 + Z^2) - Z\bar{\theta}\Gamma^{11}\dot{\theta} + \bar{\chi}_1 d_2 \right) . \tag{29}$$

We will show now that the D-0-brane action can be obtained from this one upon solving equations of motion for $\mathbf{P}_m$, V, χ_1, and d_2. Here $V(\tau)$ is a Lagrange multiplier, $Z = T$ is some constant parameter in front of the WZ term and $\mathbf{P}^2 \equiv \mathbf{P}^m\eta_{mn}\mathbf{P}^n$. The chiral spinors χ_1 and d_2 are auxiliary fields. They are introduced to close the gauge symmetry algebra off shell. To verify that this first order action is one of the D-p-brane family actions given in (1) we can use equations of motion for $\mathbf{P}_m$

$$\mathbf{P}_m = -\frac{1}{V}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta}) , \tag{30}$$

and for the auxiliary fields $\chi_1 = 0$ and $d_2 = 0$. The action (29) becomes

$$S = \int d\tau \left(-\frac{1}{2V}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2 + \frac{1}{2}VZ^2 - Z\bar{\theta}\Gamma^{11}\dot{\theta} \right) . \tag{31}$$

Equation of motion for V is

$$V^2 = -\frac{1}{Z^2}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2 , \tag{32}$$

and we can insert $V = -\frac{1}{Z}\sqrt{-(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2}$ back into the action (31) and get

$$S = -Z \left(\int d\tau \sqrt{-(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2} + Z\bar{\theta}\Gamma^{11}\dot{\theta} \right) . \tag{33}$$

This is the action (1) for D-0-brane at $T = Z$ as given in (27).

The action (29) is invariant under the 16-dimensional irreducible κ-symmetry and under the reparametrization symmetry. The gauge symmetries are (we denote $\Gamma^m\mathbf{P}_m = \not{\mathbf{P}}$):

$$\begin{aligned}
\delta\bar{\theta} &= \bar{\kappa}_2(\Gamma^{11}Z + \not{\mathbf{P}}) , && (34)\\
\delta X^m &= -\eta\mathbf{P}^m - \delta\bar{\theta}\Gamma^m\theta - \bar{\kappa}_2\Gamma^m d , && (35)\\
\delta V &= \dot{\eta} + 4\bar{\kappa}_2\dot{\theta} + 2\bar{\chi}_1\kappa_2 , && (36)\\
\delta\bar{\chi} &= \bar{\kappa}_2\dot{\not{\mathbf{P}}} , && (37)\\
\delta d &= [\mathbf{P}^2 + Z^2]\kappa_2 . && (38)
\end{aligned}$$

Here $\eta(\tau)$ is the time reparametrization gauge parameter and $\kappa_2(\tau) = \frac{1}{2}(1 - \Gamma^{11})\kappa(\tau)$ is the 16-dimensional parameter of κ-symmetry. The gauge symmetries form a closed algebra

$$[\delta(\kappa_2), \delta(\kappa_2')] = \delta(\eta = 2\bar{\kappa}_2\not{\mathbf{P}}\kappa_2') . \tag{39}$$

To bring the theory to the canonical form we introduce canonical momenta to θ and to V and find, excluding auxiliary fields

$$L = \mathbf{P}_m \dot{X}^m + P_V \dot{V} + \bar{P}_\theta \dot{\theta} + \frac{1}{2} V(\mathbf{P}^2 + Z^2) + P_V \varphi + \left(\bar{P}_\theta + \bar{\theta}(\not{\mathbf{P}} + Z\Gamma^{11}) \right) \psi \ . \tag{40}$$

We have primary constraints $\bar{\Phi} \equiv \bar{P}_\theta + \bar{\theta}(\not{\mathbf{P}} + Z\Gamma^{11}) \approx 0$ and $P_V = 0$. The Poisson brackets for 32 fermionic constraints are

$$\{\Phi, \Phi\} = 2C(\not{\mathbf{P}} + \Gamma_{11} Z) \ . \tag{41}$$

We also have to require that the constraints are consistent with the time evolution $\{P_V, H\} = 0$. This generates a secondary constraint

$$t = \mathbf{P}^2 + Z^2 \ . \tag{42}$$

Thus the Hamiltonian is weakly zero and any physical state of the system satisfying the reparametrization constraint is a BPS state $M = |Z|$ since

$$\mathbf{P}^2 + Z^2 |\Psi\rangle = 0 \qquad \Longrightarrow \qquad Z^2 |\Psi\rangle = -\mathbf{P}^2 |\Psi\rangle = M^2 |\Psi\rangle \ . \tag{43}$$

The 32×32 -dimensional matrix $C(\not{\mathbf{P}} + \Gamma_{11} Z)$ is not invertible since it squares to zero when the reparametrization constraint is imposed. This is a reminder of the fact that D-0-brane is a d=11 massless superparticle. The 32 dimensional fermionic constraint has a 16-dimensional part which forms a first class constraint and another 16-dimensional part which forms a second class constraint. We notice that the Poisson brackets reproduce the $d = 10$, $N = 2$ algebra with the central charge which can also be understood as $d = 11$, $N = 1$ supersymmetry algebra with the constant value of $\mathbf{P}_{11} = Z$.

We proceed with the quantization and gauge-fix κ-symmetry covariantly by taking $\theta_2 = 0, \theta_1 \equiv \lambda$ and find

$$L^\kappa_{g.f.} = \mathbf{P}_m (\dot{X}^m - \bar{\lambda} \Gamma^m \dot{\lambda}) + \frac{1}{2} V(\mathbf{P}^2 + Z^2) \ . \tag{44}$$

The 16-dimensional fermionic constraint

$$\bar{\Phi}_\lambda \equiv (\bar{P}_\lambda + \bar{\lambda} \not{\mathbf{P}}) \approx 0 \tag{45}$$

forms the Poisson bracket

$$\{\Phi^\alpha_\lambda, \Phi^\beta_\lambda\} = 2(\not{\mathbf{P}} C)^{\alpha\beta} \ . \tag{46}$$

The matrix $\not{\mathbf{P}} C$ is perfectly invertible as long as the central charge Z is not vanishing. The inverse to (46) is

$$\{\Phi^\alpha, \Phi^\beta\}^{-1} |_{t=0} = [2(\not{\mathbf{P}} C)^{\alpha\beta}]^{-1} = \frac{(C \not{\mathbf{P}})_{\alpha\beta}}{2\mathbf{P}^2} \ . \tag{47}$$

This proves that the fermionic constraints are second class and that the fermionic part of the Lagrangian

$$- \bar{\lambda} \not{\mathbf{P}} \dot{\lambda} \equiv -i \lambda^\alpha \Phi_{\alpha\beta} \dot{\lambda}^\beta \ , \qquad \Phi_{\alpha\beta} = -i(C \not{\mathbf{P}})_{\alpha\beta} \ , \tag{48}$$

is not degenerate in a Lorentz covariant gauge. None of this would be true for a vanishing central charge. Note that in the rest frame $\mathbf{P}_0 = M, \vec{\mathbf{P}} = 0$, hence

$$\Phi_{\alpha\beta} = M\delta_{\alpha\beta} \ . \tag{49}$$

For D-0-brane one can covariantly gauge-fix the reparametrization symmetry by choosing the $V = 1$ gauge and including the anticommuting reparametrization ghosts b, c. This brings us to the following form of the action:

$$L^{\kappa,\eta}_{g.f.} = \mathbf{P}_m \dot{X}^m - \bar{\lambda} \not{\mathbf{P}} \dot{\lambda} + \frac{1}{2}(\mathbf{P}^2 + Z^2) + b\dot{c} \ . \tag{50}$$

Now we can define Dirac brackets

$$\{\lambda, \bar{\lambda}\}^* = \{\lambda, \bar{\Phi}\}\{\bar{\Phi}, \Phi\}^{-1}\{\Phi \ , \bar{\lambda}\} = \frac{\not{\mathbf{P}}}{2\mathbf{P}^2} = -\frac{\not{\mathbf{P}}}{2Z^2} \ . \tag{51}$$

The generator of the 32-dimensional supersymmetry is

$$\bar{\epsilon}Q = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11} Z)\lambda \ . \tag{52}$$

It forms the following Dirac bracket

$$[\bar{\epsilon}Q \ , \bar{Q}\epsilon']^* = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11} Z)\frac{\not{\mathbf{P}}}{2\mathbf{P}^2}(\not{\mathbf{P}} + \Gamma^{11} Z)\epsilon' = \bar{\epsilon}\Gamma^{\hat{m}}\mathbf{P}_{\hat{m}}\epsilon' = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11} Z)\epsilon' \ . \tag{53}$$

We can also rewrite it in d=11 Lorentz covariant form

$$[\bar{\epsilon}Q \ , \bar{Q}\epsilon']^* = \bar{\epsilon}\Gamma^{\hat{m}}\mathbf{P}_{\hat{m}}\epsilon' = \bar{\epsilon}\hat{\not{\mathbf{P}}}\epsilon' \ , \qquad \hat{m} = 0, 1, \cdots, 8, 9, 10, \qquad Z = \mathbf{P}_{\hat{10}} \ , \qquad \Gamma^{11} = \Gamma^{\hat{10}} \ . \tag{54}$$

This Dirac bracket realizes the d=11, N=1 supersymmetry algebra or, equivalently, d=10, N=2 supersymmetry algebra with the central charge Z.

One can also to take into account that the path integral in presence of second class constraints has an additional term with $\sqrt{\mathrm{Ber}\{\Phi_\lambda, \Phi_\lambda\}} \sim \sqrt{\mathrm{Ber}\,\Phi_{\alpha\beta}}$ [17]. It can be used to make a change of variables

$$S_\alpha = \Phi^{1/2}_{\alpha\beta} \ \lambda^\beta \ . \tag{55}$$

The action becomes

$$L = \mathbf{P}_m \dot{X}^m - iS_\alpha \dot{S}_\alpha + b\dot{c} - H \tag{56}$$

$$H = -\frac{1}{2}(\mathbf{P}^2 + Z^2) \ . \tag{57}$$

The generators of global supersymmetry commuting with the Hamiltonian take the form

$$\bar{\epsilon}Q = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11} Z)\Phi^{-1/2} S \ . \tag{58}$$

Taking into account that $\{S_\alpha, S_\beta\}^* = -\frac{i}{2}\delta_{\alpha\beta}$ we have again realized $d = 10$, $N = 2$ supersymmetry algebra in the form (53) or (54). The nilpotent BRST operator in this gauge where only reparametrization ghosts are propagating is given by

$$Q_{BRST} = cH \qquad H = \{b, Q_{BRST}\} \qquad (Q_{BRST})^2 = 0 \tag{59}$$

and here we used the fact that $\{b, c\} = 1$. In turn, the Hamiltonian (and therefore the BRST operator) can be constructed from supersymmetry generators as follows:

$$H = \frac{1}{2}\{Q_\alpha, Q_\beta\}^* \{\bar{Q}^\alpha, \bar{Q}^\beta\}^* = \frac{1}{2}(C\hat{\not{\mathbf{P}}})_{\alpha\beta}(\hat{\not{\mathbf{P}}}C)^{\alpha\beta} = -\frac{1}{2}\hat{\not{\mathbf{P}}}^2 = -\frac{1}{2}(\not{\mathbf{P}}^2 + Z^2) \tag{60}$$

The terms with anticommuting fields S_α can be rewritten in a form where it is clear that they can be interpreted as world-line spinors,

$$L = \mathbf{P}_m \partial_0 X^m + \bar{S}_\alpha \rho^0 \partial_0 S_\alpha + b\dot{c} - H \ . \tag{61}$$

Here $\bar{S}_\alpha = i S_\alpha \rho^0$ and $(\rho^0)^2 = -1$, $\rho^0 = i$ being a 1-dimensional matrix.

Thus, we have the original 10 coordinates X^m and their conjugate momenta $\mathbf{P}_m$, and a pair of reparametrization ghosts. There are also 16 anticommuting world-line spinors S, describing 8 fermionic degrees of freedom. The Hamiltonian is quadratic. The ground state with $M^2 = Z^2$ is the state with the minimal value of the Hamiltonian. Thus for the D-superparticle one can see that the condition for the covariant quantization is satisfied in the presence of a central charge which makes the mass of a physical state non-vanishing. The global supersymmetry algebra is realized in a covariant way, as different from the light-cone gauge.

Thus we have found that covariantly quantized D-0-brane has a quadratic action with the physical state being the BPS state $M = |Z|$. The resulting supersymmetry generator is $d = 10$ Lorentz covariant and the Dirac bracket of the quantized theory form $d = 10$, $N = 2$ supersymmetry algebra with a central charge.

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